Risk Reduction Potential

Research Paper 006

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In this paper we introduce the concept of risk reduction potential. Risk reduction potential answers the question: by how much can a specific hedge reduce the risk of a portfolio?

Introduction

The defining characteristic of hedge funds is that they use hedging to reduce the risk of their investment portfolios. For example, to hedge a long position, a fund will often take a short position in a similar security. The hedge is said to be effective if the short position makes money when the long position suffers a loss. For the hedge to be effective, then, the long and short securities must be highly correlated[1].

The corollary of this fact is that you cannot hedge with a security that is uncorrelated with your existing portfolio. If you attempt to hedge with an uncorrelated security, you will end up increasing the risk of your portfolio, not lowering it.

In the extreme, if a security is 100% correlated with your existing portfolio, all risk can be hedged. If a security has 0% correlation to your existing portfolio, it cannot reduce the risk of your portfolio. In between 0% and 100%, you might expect the relationship between correlation and risk reduction potential to be linear. That is, you might expect a security with 60% correlation to be able to reduce the risk of your portfolio by 60% and a security with 30% correlation to be able to reduce the risk by 30%. This is not the case. The relationship is not linear. In reality the security with 30% correlation can barely reduce the risk of your portfolio, and the security with 60% correlation can reduce risk four times as much as the security with 30% correlation. Mathematically, the risk reduction potential (RRP) is;

$$\mathrm{RRP} = 1 - \sqrt{1 - \rho^2}$$

Equation 1

where ρ is the correlation between the portfolio and the hedging instrument.

In the following sections we provide a more specific definition for the risk reduction potential, and derive the formula given in Equation 1. Along the way, we discuss the appropriate use and limitations of the RRP equation.

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[1] If the *securities* are positively correlated then the long and short *positions* will be *negatively* correlated. Keeping the correlation of the securities and the positions straight can be confusing, but it should be clear from the context. As we will see, what we are most concerned with is not so much the sign of the correlation (+90% versus −90% correlation) but the magnitude of the correlation (±90% versus 0%). For a hedge fund, given the correlation of a security, we can always choose the sign of the correlation of the position by either taking a long or short position in the security.

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Derivation of the Formula

Throughout this paper we will assume that the relationship between the portfolio and the hedging security is linear. If R_t is the return at time t of a portfolio, and $R_{f,t}$ is the return at time t of a risk factor that we wish to hedge, then we assume that we can model the portfolio as:

$$R_t = \alpha + \beta R_{f,t} + \varepsilon_t$$

Equation 2

where, α and β are constants, and ε_t is a mean zero error term. $\beta R_{f,t}$ represents systematic risk of the portfolio, which can be hedged by shorting the risk factor, and ε_t represents idiosyncratic risk, which cannot be hedged.

In the real world, we may be interested in hedging with non-linear instruments, such as options, and our own portfolio may contain non-linear instruments. Even if we restrict ourselves to linear instruments such as equities and futures, it may not be the case that the relationship between the portfolio and the hedging security is linear over a wide range of returns. If the relationship between the portfolio and the hedging security is non-linear, then we cannot apply Equation 1 to quantify the risk reduction potential, but the general concept still holds: hedging can only be effective when the portfolio and the hedging security are highly correlated.

Assuming a linear relationship as describe in Equation 2, if we short β of the risk factor, we will be left with only the idiosyncratic risk. The return of the hedged portfolio at time t, R_t^* is:

$$R_t^* = \alpha + \varepsilon_t$$

Equation 3

The standard deviation of the hedged portfolio represents a minimum. This is the most that we can reduce the standard deviation of the portfolio using the hedging security.

The risk reduction potential (RRP) indicates how much the standard deviation of a portfolio can be reduced by hedging out the systematic component of its risk. It varies from 0% (risk cannot be reduced), to 100% (all risk can be hedged). Let σ_R denote the standard deviation of the portfolio before hedging; that is the standard deviation of the portfolio as described in Equation 2. Let σ_{R^*} denote the standard deviation of the portfolio after hedging, the standard deviation of the portfolio as describe by Equation 3. The RRP is defined as:

$$\mathrm{RRP} = -\frac{\Delta\sigma}{\sigma_R} = -\frac{\sigma_{R^*} - \sigma_R}{\sigma_R} = \frac{\sigma_R - \sigma_{R^*}}{\sigma_R} = 1 - \frac{\sigma_{R^*}}{\sigma_R}$$

Equation 4

For an ordinary least squares (OLS) regression $R_{f,t}$ and ε_t should be independent, therefore

$$\sigma_R^2 = \beta^2\sigma_f^2 + \sigma_\varepsilon^2$$
$$\sigma_{R^*}^2 = \sigma_\varepsilon^2$$

Equation 5

Where σ_f and σ_ε are the standard deviation of the factor returns and the error term, respectively. If ρ is the correlation between R_t and $R_{f,t}$, then we also know that

$$\beta = \rho \frac{\sigma_R}{\sigma_f}$$

Equation 6

Combining with Equation 5 we have

$$\begin{aligned} \sigma_R^2 &= \beta^2 \sigma_f^2 + \sigma_\varepsilon^2 \\ &= \rho^2 \sigma_R^2 + \sigma_\varepsilon^2 \end{aligned}$$

Equation 7

Rearranging terms

$$\sigma_\varepsilon^2 = (1 - \rho^2)\sigma_R^2$$

Equation 8

Substituting back into our equation for RRP:

$$\begin{aligned} \mathrm{RRP} &= 1 - \frac{\sigma_{R^*}}{\sigma_R} \\ &= 1 - \sqrt{\frac{\sigma_{R^*}^2}{\sigma_R^2}} \\ &= 1 - \sqrt{\frac{\sigma_\varepsilon^2}{\sigma_R^2}} \\ &= 1 - \sqrt{\frac{(1 - \rho^2)\sigma_R^2}{\sigma_R^2}} \\ &= 1 - \sqrt{1 - \rho^2} \end{aligned}$$

Equation 9

This is the equation for risk reduction potential that we presented in the introduction.

Understanding the Formula

Exhibit 1 shows a graph of correlation versus risk reduction potential. First notice that the graph is symmetric. We can get the same risk reduction using a security that is −50% correlated with the existing portfolio as we can using a security that is +50% correlated, we simply take a long position in the security instead of a short position. The second thing to notice is that the relationship is non-linear. From −50% to +50% the RRP is very low, there is very little reduction in risk. Beyond ±50% RRP increase rapidly.

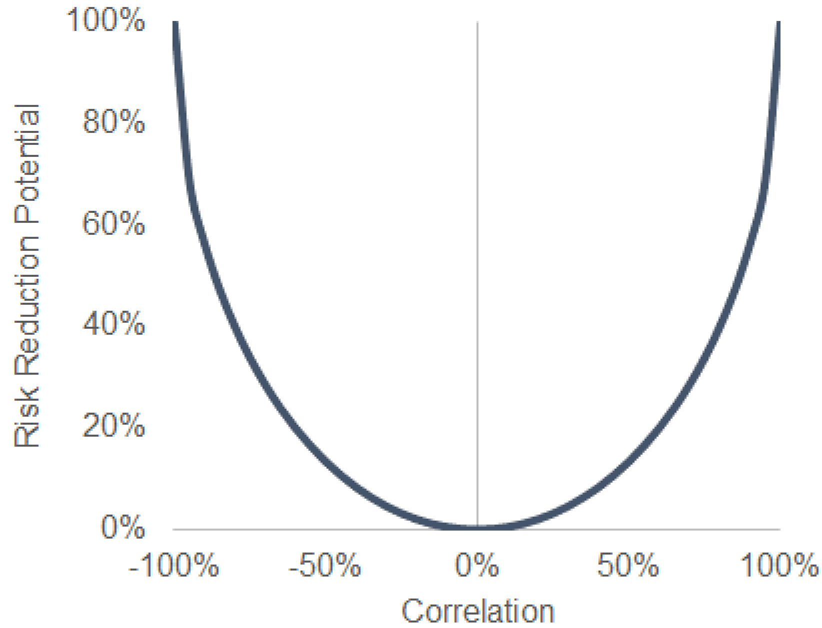

Exhibit 1: Relationship between Correlation and RRP

For portfolio managers the non-linear relationship between correlation and RRP means that low correlation hedges are likely to be considerably less effective than your intuition suggests. It also means that the reward for finding hedges with slightly higher correlations can be significant.

Conclusion

RRP is a convenient way to understand the potential effectiveness of different hedging instruments. If we attempt to hedge using instruments with low correlation to the existing portfolio, we will not reduce the standard deviation of the portfolio significantly. In order to dramatically reduce the risk of a portfolio we need to hedge with instruments which are highly correlated with our existing portfolio.

