Volatility Surface Stress Tests

Research Paper 008

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Volatility Surface Stress Tests

Term Structure Stress Test

In the term structure stress test, we adjust the implied volatility of every option based on the time to expiration. We then use this stress volatility to calculate the change in portfolio value. In a steepening scenario, the implied volatility of long-dated options increases, and the implied volatility of near-dated options decreases. The opposite is true in the flattening scenarios.

More specifically, given the current implied volatility, σ, the implied volatility in the stress scenario is set to

$$\sigma_{\text{stress}} = \sigma[1 + \beta_\tau(\tau - \alpha_\tau)]$$

where α_τ and β_τ are two arbitrary constants. α_τ can be thought of as the pivot point. If β_τ is positive, then options with time to expiration greater than α_τ will have higher implied volatility, and options with time to expiration lower than α_τ will have lower implied volatility. β_τ determines the magnitude of the adjustment. As an example, if $\alpha_\tau = 0.25$ and $\beta_\tau = 1.0$ then an option with 6 months to expiration and with a current implied volatility of 20% will have a stress volatility of 25%:

$$\sigma_{\text{stress}} = 0.20[1 + 1.0(0.50 - 0.25)] = 0.20[1.25] = 0.25$$

whereas an option with a current implied volatility of 20% but with just 1 month until expiration would have a stress implied volatility of approximately 17%:

$$\sigma_{\text{stress}} = 0.20[1 + 1.0(0.08 - 0.25)] = 0.20[0.83] = 0.17$$

Figure 1 shows an example of a stress to an already upward sloping term structure.

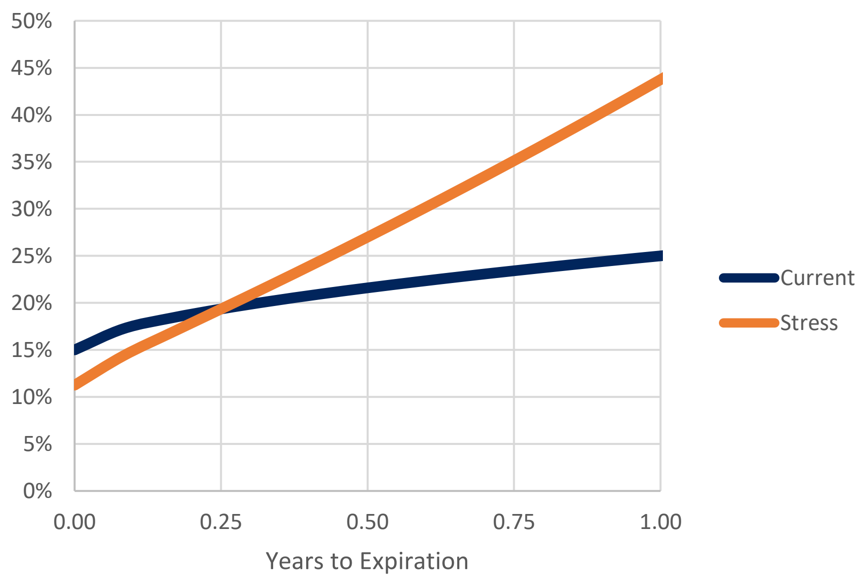

Figure 1: Term Structure Stress, $\alpha_\tau = 0.25$ and $\beta_\tau = 1.00$

Smile Stress Test

The smile stress test is similar to the term structure stress test, only rather than being based on the time to expiration it is based on the option's delta. Mathematically, for a call option

$$\sigma_{\text{stress}} = \sigma[1 + \beta_\Delta(\Delta_C - \alpha_\Delta)]$$

where Δ_C is the Black-Scholes-Merton delta for a call option or its equivalent for non-European options. For a put option, we use the call-equivalent delta, that is $\Delta_C = \Delta_P + 1$.

For a smile that is downward sloping in terms of strike, such as we typically see equity index markets, a positive β_Δ will produce a steeper smile in the stress scenario. Figure 2 shows a stress test where the pivot is set to 0.50 and $\beta_\Delta = 0.80$.

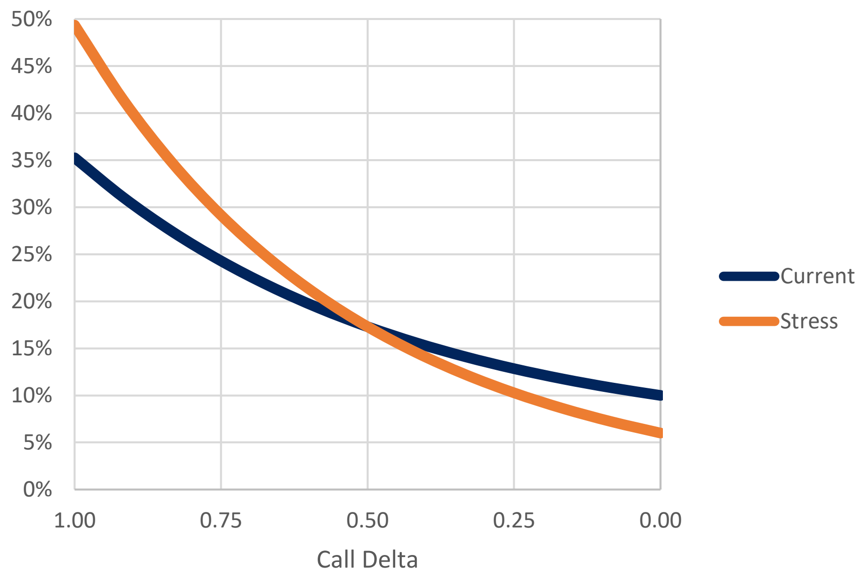

Figure 2: Smile Stress, $\alpha_\Delta = 0.50$ *and* $\beta_\Delta = 0.80$

Combined Term Structure and Smile Stress Test

The term structure stress and smile stress can be combined as follows

$$\sigma_{\text{stress}} = \sigma[1 + \beta_\tau(\tau - \alpha_\tau)][1 + \beta_\Delta(\Delta_C - \alpha_\Delta)]$$

In either the term structure, smile, or combined stress tests, if the betas are set too high or too low this could produce negative implied volatilities. In practice for typical options and reasonable betas this is rarely an issue. Just in case, by default we restrict any implied volatilities to a minimum of 0.10%.

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