Constant Risk Portfolios

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Many managers will lower their exposure to a security when that security becomes more volatile. For example, if you are only comfortable holding $200 of XYZ when its standard deviation is 8%, you might decrease your holdings to $100 of XYZ when its standard deviation increases to 16%. In the short run, this strategy may help a manager to avoid large losses. In the long run, this type of behavior can also significantly improve a manager's risk adjusted performance. To better understand this strategy, we examine constant risk portfolios, where the overall risk of a portfolio is kept constant by systematically adjusting the exposure of the portfolio.

Maintaining Constant Risk

As the example in the introduction suggests, if we want the P&L standard deviation due to a certain risk factor to be constant then we should cut our exposure to that factor in half when the standard deviation of that risk factor doubles. Looked at another way, we want to keep the product of the exposure and the factor standard deviation constant. So, if we are comfortable with and exposure of $100 when standard deviation is 10%, then we should also be comfortable with $200 and 5%, and $50 and 20%. In each case the product of the exposure and the factor standard deviation is $10. Looked at another way, if we follow this rule, a one standard deviation move in the factor will always result in $10 of P&L.

In this paper, we are going to focus on one risk factor, the market risk to a portfolio from its exposure to the S&P 500 as represented by its beta exposure to the index. If the beta exposure of the portfolio is 50% and the S&P 500 has a return of 2%, then the portfolio P&L will be 1% = 50% x 2%. We could estimate the standard deviation of the S&P 500 using historical data, but that would complicate matters by introducing a model to estimate standard deviation. Instead, we will use the VIX as our estimate of S&P 500 standard deviation.[1] At time, *t*, given the beta of our portfolio to the S&P 500, β_t, and the level of the VIX, V_t, we want

$$\beta_t V_t = c$$

Equation 1

where *c* is some arbitrary constant. Once we have determined *c*, we can turn this equation into a rule for managing our exposure,

$$\beta_t = \frac{c}{V_t}$$

Equation 2

How do we determine *c*? One way to determine *c* would be to choose an arbitrary level where we are comfortable. If we are comfortable with $\beta_t = 20\%$ when $V_t = 16\%$ then we could set *c* = 3.2%. In this

[1] For various reasons, the VIX tends to be slightly higher on average than realized standard deviation. We could easily adjust our rule to take into account this bias, but, as we will see, none of our results would be altered by making this adjustment. For the rest of the paper then, without loss of generality, we are going to act as if the VIX is an unbiased predictor of S&P 500 standard deviation.

paper, to make comparison with a constant beta exposure portfolio easier, we are going set *c* so that the mean exposure is equal to some arbitrary value $\bar{\beta}$,

$$\frac{1}{T}\sum_{t=1}^{T}\beta_t = \bar{\beta}$$

Equation 3

Combining Equation 1 and Equation 3, we have

$$c = \bar{\beta}\frac{T}{\sum_{t=1}^{T}\frac{1}{V_t}}$$

Equation 4

In other words, we will set the *c* equal to $\bar{\beta}$ multiplied by the harmonic mean of the level of the VIX.

In our analysis in this paper, we are going to focus on the 20-year period from April 1998 through March 2018, during which time the harmonic mean of the VIX was 17.68, and we are going to target a mean beta exposure of 20%. Our constant would then be *c* = 20% x 17.68 = 3.54. As an example, if the VIX is equal to 16, using Equation 2, we should set our exposure equal to 22.1%,

$$\beta_t = \frac{c}{V_t} = \frac{3.54}{16} = 22.1\%$$

Equation 5

Transaction Costs

In practice, we cannot continuously adjust our portfolio exposures. To make our analysis realistic, we will assume that the portfolio is rebalanced at most daily. We will also look at the effectiveness of less frequent rebalancing. In one scenario, rather than rebalancing daily, we rebalance weekly. We also look at scenarios where we rebalance only when the product $\beta_t V_t$ is either 5% or 10% different from the target value. This is similar to delta hedging an option portfolio using differences in the current and target delta (see Derman and Miller, 2016, Chapter 7). In all cases we will assume that rebalancing, when it happens, happens at the end of the day, based on the closing level of the VIX. We will consider scenarios with no transaction costs, and scenario where there is a variable transaction cost equal to 0.10% of the absolute value traded.

To make the final results more meaningful, we also assumed the portfolio manager generates 3 bps of alpha per day. This brings the cumulative returns for each strategy roughly in line with the cumulative return for the S&P 500 over the sample period.

Results

Table 1 presents the results for five different strategies without fees along with the returns for the S&P 500 (SPX). Each of the five strategies targets a 20% mean beta exposure. They only differ in how often they rebalance. The five strategies in the table are labeled as,

1. **Const β**: beta exposure reset to 20% at the end of each day

2. **Daily**: portfolio is rebalanced daily
3. **Weekly**: portfolio is rebalanced weekly
4. **Five**: portfolio is rebalanced whenever $\beta_t V_t$ is 5% different from the target value
5. **Ten:** portfolio is rebalanced whenever $\beta_t V_t$ is 10% different from the target value

The rows of the table show, the number of times each strategy had to be rebalanced over 20 years, the mean annualized return, the annualized standard deviation, the worst single day over the 20-year period, the worst month, and the Sharpe ratio assuming a risk-free rate of 2% per annum.

			Constant Risk			
	SPX	Const β	Daily	Weekly	Five	Ten
Rebals		5,218	5,218	1,043	1,777	739
Mean	6.43%	9.82%	9.48%	9.62%	9.47%	9.52%
Std. Dev.	19.30%	3.86%	2.82%	2.87%	2.81%	2.81%
Worst Day	-9.03%	-1.78%	-1.07%	-1.17%	-1.07%	-1.09%
Worst Month	-16.79%	-2.50%	-1.08%	-1.15%	-1.13%	-1.06%
Sharpe(rfr=2%)	0.23	2.03	2.65	2.65	2.65	2.68

Table 1: April 1998 - March 2018, No Fees

As can be seen, maintaining a constant risk level significantly improves performance. Compared to the constant beta exposure strategy, the worst day and month are not as bad, and the Sharpe ratio is higher. Rebalancing less frequently does not have a significant impact on performance.

Table 2 shows the results over the same period, only this time we have included the 0.10% trading fee. It doesn't take that much trading to maintain a constant risk level. When we include the fees, the returns and Sharpe ratios are lower but not significantly lower. The constant risk strategies still significantly outperform the strategy of maintain a constant beta. Though the differences are small, of the four constant risk strategies, the strategy of rebalancing only when the portfolio's risk level is more than 10% off target has the highest Sharpe ratio.

			Constant Risk			
	SPX	Const β	Daily	Weekly	Five	Ten
Rebals		5,218	5,218	1,043	1,777	739
Mean	6.43%	9.79%	9.25%	9.53%	9.30%	9.42%
Std. Dev.	19.30%	3.86%	2.82%	2.87%	2.82%	2.81%
Worst Day	-9.03%	-1.78%	-1.08%	-1.17%	-1.08%	-1.10%
Worst Month	-16.79%	-2.51%	-1.09%	-1.16%	-1.14%	-1.07%
Sharpe(rfr=2%)	0.23	2.02	2.57	2.62	2.59	2.64

Table 2: April 1998 - March 2018, with Fees

If trading cost are higher, or if the volatility of standard deviation is great, a constant risk strategy may be more negatively impacted.

But Isn't This Just 2008

Moving from a constant beta exposure to a constant risk exposure appears to significantly improve performance even with trading fees. The constant risk strategy produces a higher Sharpe ratio and the worst day and worst month are not nearly as bad. That said, our data set spans the 2008 financial crisis, a period when the VIX was extremely high and market returns were extremely negative. If you had followed a constant risk strategy in 2008, you would have significantly lowered your market exposure and significantly reduced your losses. We might wonder then, if 2008 doesn't explain all of the differences.

Table 3 shows the results of the same strategies, only for the period post-2008. With the exception of the 2018 partial year, the market was up every year during this period, but the constant risk strategies still outperformed. The constant risk strategies don't require a crisis to outperform.

			Constant Risk			
	SPX	Const β	Daily	Weekly	Five	Ten
Rebals		5,218	5,218	1,043	1,777	739
Mean	6.56%	5.10%	4.79%	4.91%	4.82%	4.90%
Std. Dev.	11.34%	2.27%	1.84%	1.89%	1.84%	1.83%
Worst Day	-6.65%	-1.30%	-0.82%	-1.02%	-0.82%	-0.82%
Worst Month	-10.65%	-1.56%	-0.82%	-1.01%	-0.82%	-0.78%
Sharpe(rfr=2%)	0.40	1.37	1.52	1.54	1.54	1.59

Table 3: January 2009 - March 2018, with Fees

Why this Works

So why do the constant risk strategies perform so well? It easy to understand why the worst day and worst month are not as bad for constant risk strategies. The most significant drops in the S&P 500 are more likely to happen when the VIX is high. By lowering exposures when the VIX is high, the constant risk strategies significantly reduce the impact of big drops in the S&P 500.

What may be less obvious is that by reducing the volatility of volatility, the constant risk strategy reduces long-term volatility and improves risk-adjusted performance. To see why this works, consider a fund that has a return of 10% with a standard deviation of 10% one year, followed by a return of 20% with a standard deviation of 20% the next year. If the risk-free rate is 0%, then the Sharpe ratio in each year is 1,

$$S_1 = \frac{10\% - 0\%}{10\%} = 1 \quad , \quad S_2 = \frac{20\% - 0\%}{20\%} = 1$$

You might think that the Sharpe ratio over the two-year period would also be equal to 1, but it is actually less than 1. The mean return over the two year period is very close to 15%, as we would expect, $[(1 + 10\%)(1 + 20\%)]^{1/2} - 1 = 14.89\%$. The standard deviation, however, is quite a bit higher than 15%,

$$\sigma_{1+2} = \sqrt{10\% \times 10\% + 20\% \times 20\%} = 15.81\%$$

The Sharpe ratio is then

$$S_{1+2} = \frac{14.89\%}{15.81\%} = 0.94$$

If the return and standard deviation in the second period had been 30% rather than 20%, the reduction in the two-year Sharpe ratio would have been even greater. In general, all else being equal, greater variability in the standard deviation of returns leads to a higher long-term standard deviation, and a lower Sharpe ratio.

The strategy may also improve performance because of compounding. A portfolio that has a −10% return, followed by a +10% return, has a −1% cumulative return. A portfolio that has a −50% return, followed by a +50% return, has a −25% cumulative return. Even though the mean of the two returns is zero in both scenarios, the second scenario with more volatile returns has a lower cumulative return. For this reason, unless mean returns are significantly higher when returns are more volatile, then lowering exposure when volatility increases should improve cumulative returns. If increased volatility is associated with lower return on average, then the constant risk strategy will improve performance even more.

Options?

If risk increases gradually over time, a constant risk strategy will gradually reduce exposures as market risk increases, thereby avoiding large losses. But what if risk increases suddenly? In that case, the constant risk strategy will not have time to adjust. In the worst-case scenario, when market risk is below average prior to the event, the constant risk strategy will actually have higher than normal exposures going into the event and will suffer worse losses relative to a constant beta exposure strategy.

One possible way to guard against these sudden market drops is to buy out-of-the-money puts. Conveniently, when following a constant risk strategy, portfolio exposures will be highest when volatility is low, and options are cheap. Similarly, when market volatility is high, and these options are expensive, the constant risk strategy eliminates the need for buying options by reducing exposures.

Multiple Risk Factors

In this paper, we have focused on one factor, broad stock market risk represented by the S&P 500. The same logic can easily be applied to other risk factors and can even be applied to multiple risk factors within the same portfolio.

Conclusions

In this paper we have demonstrate how maintaining a more constant risk level, by varying factor exposures over time, can reduce the probability of large losses and improved risk-adjusted performance. Trading costs are likely to negatively impact overall returns, but the impact is likely to be small for reasonable volatility and reasonable trading costs. A constant risk strategy is likely to underperform when faced with a sudden unexpected drop in the market, but these losses could potentially be mitigated by buying out-of-the-money puts.

References

Derman, Emanuel and Michael B. Miller. 2016. *The Volatility Smile*. Hoboken, NJ: John Wiley & Sons.

Disclaimer

The results presented in this paper are based on historical data and includes various assumptions. Past performance is no guarantee of future results. Under no circumstances should this paper be construed as offering investment advice.

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