Additive Profit Calculation

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Standard Return Calculations

In the Northstar Risk/Performance Application there are two percent return fields, P&L(%) and P&L(%,+). The first is the standard cumulative return. At each level —portfolio, sub-portfolio, and security— the same standard formula is applied. At a given level, if the return in the first period is R_1 and the return in the second period is R_2, then the cumulative return, R_C, over both periods is

$$R_C = (1 + R_1)(1 + R_2) - 1$$

The problem with the standard cumulative return is that the sum of the parts is not equal to the whole, if we add up the standard cumulative returns for all of the securities in a portfolio it will not equal the cumulative return of the portfolio itself.

Additive Profit Calculation

The standard cumulative return can be useful for evaluating positions and sub-portfolios in isolation, but to evaluate the contribution to overall performance it is often preferable to have a cumulative return that is additive. The P&L(%,+) field is calculated in such a way that it is additive. At the portfolio level, P&L(%) and P&L(%,+). For sub-portfolios and positions within the portfolio P&L(%,+) makes an adjustment to P&L(%) so that the sum of the parts is equal to the total.

To understand the basic problem with the standard method and the adjustment, assume that we have a portfolio, *P*, with two positions, *A* and *B*. We start by introducing some notation:

A_1 = percent return, relative to fund AUM, in period 1 for *A*

A_2 = percent return, relative to fund AUM, in period 2 for *A*

A_C = percent return, relative to fund AUM, for A, standard compounding

B_1 = profit in period 1 for *B*

$P_1 = A_1 + B_1$ = profit in period 1 for the portfolio

Then the standard cumulative return of *A* and *B* are then

$$\begin{aligned} A_C &= (1 + A_1)(1 + A_2) - 1 \\ &= A_1 + A_2 + A_1A_2 \end{aligned}$$

$$\begin{aligned} B_C &= (1 + B_1)(1 + B_2) - 1 \\ &= B_1 + B_2 + B_1B_2 \end{aligned}$$

And the standard cumulative return of the portfolio is

$$\begin{aligned} P_C &= (1 + P_1)(1 + P_2) - 1 \\ &= (1 + A_1 + B_1)(1 + A_2 + B_2) - 1 \\ &= A_C + B_C + A_1B_2 + B_1A_2 \end{aligned}$$

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Notice that the sum of the parts is not equal to the whole:

$$P_C \neq A_C + B_C$$

The cumulative return of the portfolio contains two additional terms, A_1B_2 and B_1A_2. To make the cumulative return additive, we need to distribute this additional P&L. There are an infinite number of ways that we could do this; we could allocate all of the additional P&L to *A* or all of it to *B*, and everything in between. While there is no "right" approach, one obvious solution is to allocate one of the additional terms to *A* and one to *B*. There are two ways we could do this: we could allocate A_1B_2 to *A* and B_1A_2 to *B*; or we could allocate A_1B_2 to *B* and B_1A_2 to *A*. We feel that both methods are valid and the difference in practice between the two methods is small, but we have chosen to use the first method when calculating P&L(%,+).

Additive method #1, or P&L(%,+), is then

$$A_C^* = A_C + A_1B_2$$
$$B_C^* = B_C + B_1A_2$$

Equation 1: P&L(%)

Additive method #2 is

$$\hat{A}_C = A_C + B_1A_2$$
$$\hat{B}_C = B_C + A_1B_2$$

In both cases, for each position, we can rewrite the equations to define the adjusted cumulative return solely in terms of the position's own returns and cumulative return and the cumulative return of the portfolio. For example, for method #1:

$$A_C^* = A_C + A_1(P_2 - A_2)$$
$$B_C^* = B_C + B_1(P_2 - B_2)$$

Equation 2: P&L(%), alternative formulation

Calculating the adjusted cumulative returns in this way is computationally efficient. More importantly, each term on the right-hand side is invariant to inflows/outflows, how the portfolio is grouped and to periodicity (using daily, monthly, or annual returns). This means that the adjusted cumulative returns calculated using these equations will also be invariant to flows, grouping and periodicity. Other adjustment methods either struggle with inflows/outflows, or produce different results depending on how the portfolio is grouped. For these reasons, we believe that P&L(%,+), as defined here, is an appropriate method for reporting additive cumulative returns.

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